Inflation and the Primordial Perturbation Spectrum

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Introduction

The theory of cosmic inflation is the leading hypothesis for the origin of structure in the universe. It is thought that quantum fluctuations during inflation are responsible for the primordial perturbations which grew by gravitational instability into the large-scale structure we see today. Observations of primordial scalar perturbations of the cosmic microwave background (CMB) have confirmed inflationary predictions, and the recent claim of the discovery of primordial tensor perturbations in the CMB promises to further validate inflationary theories.

Issues in Big Bang Cosmology

Historically, however, inflation was not introduced as a theory of the origin of structure, but as a theory of initial conditions to solve three issues with Big Bang cosmology: the horizon problem, the flatness problem, and the relic problem.

The horizon problem comes about if the period of radiation-domination in the early universe is taken to hold at all times before matter-radiation equality. Then, radiation-domination implies that the universe has a finite age, and thus, there is a finite particle horizon – a maximum distance over which particles can travel in the history of the universe. This horizon is much smaller than the size of the surface of last scattering from which the CMB is emitted. But the

CMB is close to isotropic despite the fact that the entirety of the surface of last scattering could not be causal contact, giving the horizon problem.

The flatness problem arises from the evolution of the density parameter under radiation-domination and matter-domination. In these periods, the comoving Hubble distance always grows, implying that the density parameter's deviation from unity always grows. Observations of the CMB and of distances to type Ia supernovae tell us that the universe was nearly flat at last scattering, and is nearly flat today. Thus, in the early universe, the density parameter must have been even closer to unity than it is today. The flatness problem asks why the universe starts with such a tuned density parameter.

Finally, the relic problem concerns itself with unwanted relics from Grand Unified Theories and theories of quantum gravity. If radiation-domination holds from the start of the universe, the Big Bang begins at temperatures higher than the phase transition temperatures of these theories, producing relics that quickly come to dominate the composition of the universe and upset the predictions of Big Bang cosmology.

Inflation as a theory of initial conditions

Inflationary theories share the property that the radiation-dominated period of Big Bang cosmology is preceded by an inflationary period where the expansion of the universe accelerates. The horizon problem is solved because the particle horizon grows rapidly during inflation. The flatness problem is solved as the comoving Hubble distance shrinks during inflation. Finally, the density of unwanted relics is diluted by this accelerating expansion. In order for an inflationary period to take place, a component with equation of state $w < \frac{1}{3}$ must dominate the universe.

While matter and radiation cannot satisfy this equation of state, (Linde, 1974) found that scalar fields could provide a component with equation of state $w < \frac{1}{3}$. (Starobinsky, 1980) then put forward an inflationary model of the early universe, but did not address the horizon, flatness, and relic problems. (Guth, 1981) proposed an inflationary model now called “old inflation” which solved the horizon, flatness, and relic problems by using a supercooled false vacuum state of the inflaton field to drive inflation. Unfortunately, his model suffered the “graceful exit” problem – at the end of inflation, the universe would either be highly inhomogeneous or highly underdense with open curvature (Guth & Weinberg, 1983) (Hawking, et al., 1982). “New inflation” or “slow-roll inflation” (Linde, 1982) (Albrecht & Steinhardt, 1982) instead drives inflation by the slow roll of the inflaton field away from the false vacuum towards true vacuum, and does not suffer the graceful exit problem.

Inflation as a theory of the origin of structure

Although historically motivated as a theory of initial conditions, it was soon realized that inflation provided an explanation for the origin of structure. In inflation, quantum fluctuations in the inflaton field follow the expansion of the universe, growing larger than the Hubble scale cH^{-1} (often called the “horizon”). Once larger than the horizon, these perturbations are frozen in since causal physics cannot change them. Slow-roll inflation predicts nearly scale-invariant perturbation spectra because perturbations on smaller spatial scales leave the horizon later, but inflationary conditions do not change much as time progresses. The scalar perturbations created in this way are the density perturbations that seed structure; tensor perturbations created in the same way are observable as primordial gravitational waves. The inflationary model of (Starobinsky, 1980) indeed predicted a scale-invariant spectrum of gravitational waves, and (Mukhanov & Chibisov, 1981) showed that Starobinsky's model produced a scale-invariant spectrum of scalar perturbations which could seed structure formation. After the introduction of slow-roll inflation, (Hawking, 1982) (Starobinsky, 1982) (Guth & Pi, 1982) (Bardeen, et al., 1983) (Mukhanov, 1985) calculated how the amplitude and spectrum of scalar perturbations depends on the slow-roll parameters.

This work will go through this calculation of the scalar perturbation spectrum created by slow-roll inflation in order to elucidate the physical principles involved. Before considering quantum fluctuations, the classical behaviour of perturbations in a homogeneous universe must be understood. In considering a perturbed universe, the curvature perturbation is introduced to characterize the scalar perturbations created by inflation. This gauge-invariant quantity is conserved on superhorizon scales for adiabatic density perturbations. Thus, the curvature perturbation that inflation creates can be calculated and propagated forward on superhorizon scales even after inflation ends. The equations of motion for classical perturbations can then be quantized to find the quantum fluctuations. In solving the horizon problem, inflation gives a homogenous universe by enforcing causal contact: but the classical behaviour of perturbations determines the quantum fluctuations that come about. It is found that these fluctuations are indeed caught up in the inflationary expansion: the power of comoving modes tends to a constant after exiting horizon. Their amplitude reflects the energy scale of inflation, and their spectrum reveals the slowly changing conditions as smaller spatial scale modes exit horizon later during inflation.

Calculating the Initial Perturbation Spectrum

This section follows (Baumann, 2009). c, $\hbar$, and $8\pi G$ are set to unity.

The Curvature Perturbation

In a perturbed universe, the split into homogeneous background and perturbation cannot be made uniquely; instead, it depends on the choice of coordinates, that is, the choice of gauge. A gauge is just a choice of timelike threads which define constant position and a choice of spacelike slices which define constant time. In a homogeneous universe, the usual FLRW position coordinates $\boldsymbol{x}$ correspond to threads which follow free-falling observers that see the universe's expansion as isotropic. With the usual slicing with cosmic time t, the universe's density evolves with time, but is homogeneous at each time. However, with a different slicing, fictitious density perturbations are created. Similarly, in a perturbed universe, a slicing can be chosen to hide density perturbations. Under these gauge changes, the change in the apparent density perturbations is compensated by other changes in the pressure, momentum density, and metric perturbations. By considering gauge-invariant combinations, fictitious perturbations can be distinguished from true perturbations: a homogeneous universe is always homogeneous in gauge-invariant quantities and perturbed universes have gauge-invariant perturbations that cannot be hidden.

In general, scalar perturbations about a homogeneous universe filled with perfect fluid can be described by perturbations in the fluid quantities density $\delta\rho$, pressure δp, and momentum density δq, as well as perturbations in the metric Φ, Ψ, B, and E:

$$ds^2 = -(1+2\Phi)dt^2 + 2a\partial_i B dx^i dt + a^2\left((1-2\Psi)\delta_{ij} + 2\delta_{ij}E\right)dx^i dx^j$$

with the Einstein equation relating the fluid perturbations to the metric perturbations, reflecting how matter and spacetime react to each other.

A useful gauge-invariant quantity for characterizing scalar perturbations during inflation is the curvature perturbation $\mathcal{R}$ (Bardeen, 1980). The curvature perturbation measures the spatial curvature on comoving slices – slices where there is no energy flux. During inflation, these slices correspond to slices of constant value of the inflaton field ϕ.

$$\mathcal{R} = \Psi - \frac{H}{\bar{\rho} + \bar{p}}\delta q$$

which during inflation becomes:

$$\mathcal{R} = \Psi + \frac{H}{\dot{\bar{\phi}}}\delta\phi$$

The Einstein equations determine the evolution of the curvature perturbation. When considering modes much larger than the horizon:

$$\dot{\mathcal{R}} = -\frac{H}{\bar{\rho} + \bar{p}}(\delta p - \frac{\dot{\bar{p}}}{\dot{\bar{\rho}}}\delta\rho)$$

If the pressure is only a function of density and not a function of entropy (ie. $p = p(\rho)$), then the resulting perturbations are adiabatic, so there is no entropy perturbation and $\delta p - \frac{\dot{\bar{p}}}{\dot{\bar{\rho}}}\delta\rho = 0$ is satisfied. Thus, for adiabatic perturbations, $\mathcal{R}$ is constant on superhorizon scales. Single slow-roll inflation has an equation of state $p \approx -\rho$, so the perturbations created during inflation are adiabatic and remain so even after the inflaton decays.

The Action Principle

A scalar field minimally coupled to gravity is governed by the action:

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right)$$

where the usual Einstein-Hilbert action has a canonical kinetic term and a potential term added. The stress-energy tensor can be found by varying the action with respect to the metric $T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g_{\mu\nu}}$, while the equation of motion can be found by varying the action with respect to the inflaton field $\frac{\delta S}{\delta\phi} = 0$. Assuming a homogeneous universe, that is, an FLRW metric and homogeneous ϕ yields the usual results for inflation:

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

This action can then be perturbed in order to find the classical evolution of perturbations during inflation. The remainder of this calculation closely follows the formulation from (Maldacena, 2003). Choosing a gauge where $\delta\phi = 0$, the resulting metric perturbation is $\delta g_{ij} = a^2(-2\mathcal{R}\delta_{ij} + h_{ij})$. Putting these perturbations into the action, the lowest order term in $\mathcal{R}$ is second order:

$$\delta S = \frac{1}{2}\int d^4x\, a^3\frac{\dot{\phi}^2}{H^2}\left(\dot{\mathcal{R}}^2 - \frac{(\partial_i\mathcal{R})^2}{a^2}\right)$$

Where we can introduce the Mukhanov variable v:

$$v = z\mathcal{R} \quad z^2 = a^2\frac{\dot{\phi}^2}{H^2}$$

and transform to conformal time τ, with primes denoting conformal time derivatives, to yield:

$$\delta S = \frac{1}{2}\int d\tau d^3x \left((v')^2 + (\partial_i v)^2 + \frac{z''}{z} v^2 \right)$$

Now we take the Fourier transform of v with respect to comoving wavenumber k in order to follow modes as they are carried by the expansion:

$$v_{\boldsymbol{k}}'' + \left(k^2 - \frac{z''}{z} \right) v_{\boldsymbol{k}} = 0$$

Recall that the curvature perturbation arises from a gauge transformation that hides the inflaton field perturbation $\delta\phi$; thus, $\mathcal{R}$ contains information about both perturbations of the inflaton field and perturbations of the metric (coupled by the Einstein equation). This differential equation now describes the classical evolution of perturbations during inflation through the Muhaknov variable v, which is just a change of variables from the curvature perturbation $\mathcal{R}$.

Quantization

Now that we have derived the classical behaviour of inflationary perturbations from the action principle, we can quantize the system to find the quantum perturbation. To do so, the field v is promoted to an operator, with its Fourier components being decomposed with the ladder operators and mode functions $v_{\boldsymbol{k}}(\tau)$:

$$\hat{v}_{\boldsymbol{k}} = v_{\boldsymbol{k}}(\tau)\hat{a}_{\boldsymbol{k}} + v_{-\boldsymbol{k}}^{*}(\tau)\hat{a}_{-\boldsymbol{k}}^{\dagger}$$

ensuring that $\hat{v} = \frac{1}{(2\pi)^3}\int d^3k\ \hat{v}_{\boldsymbol{k}}$ is Hermitian. Replacing the classical $v_{\boldsymbol{k}}$ with its operator $\hat{v}_{\boldsymbol{k}}$ shows that the mode functions $v_{\boldsymbol{k}}(\tau)$ obey the same differential equation as the Fourier components $v_{\boldsymbol{k}}$ of the classical field. Then, the momentum of the field, v', has the operator:

$$\hat{v}'_{\boldsymbol{k}} = v'_{\boldsymbol{k}}(\tau)\hat{a}_{\boldsymbol{k}} + v^{*}_{-\boldsymbol{k}}{}'(\tau)\hat{a}^{\dagger}_{-\boldsymbol{k}}$$

So the commutator of the field operator and its momentum operator is:

$$[\hat{v}_{\boldsymbol{k}}, \hat{v}'_{\boldsymbol{k}}] = i\left(v^{*}_{\boldsymbol{k}}(\tau)v'_{\boldsymbol{k}}(\tau) - v^{*}_{\boldsymbol{k}}{}'(\tau)v_{\boldsymbol{k}}(\tau)\right)\left[\hat{a}_{\boldsymbol{k}}, \hat{a}^{\dagger}_{\boldsymbol{k}}\right]$$

Imposing the canonical position-momentum commutation relation $[\hat{v}, \hat{v}^{\dagger}] = i$ and the standard commutator for the raising and lowering operators $[\hat{a}, \hat{a}^{\dagger}] = 1$ requires the mode functions $v_{\boldsymbol{k}}$ to be normalized, giving one boundary condition on the differential equation for $v_{\boldsymbol{k}}$. The other boundary condition comes from specifying the vacuum state $\hat{a}_{\boldsymbol{k}}|0\rangle = 0$. In analogy with the quantum harmonic oscillator, we want to require the vacuum to be the ground state: the lowest energy eigenstate of the Hamiltonian. There is not a unique choice, but the standard choice is to take the limit of the infinite past when all comoving scales k are well inside the Hubble horizon. Well within the horizon, any scale k can be treated as if in Minkowski spacetime, where there is a unique vacuum choice, specifying the second boundary condition.

For a specific inflationary theory, this differential equation can be solved numerically, using the evolution of $z^2 = a^2\frac{\dot{\phi}^2}{H^2}$ as determined by the evolution of the homogeneous inflaton field. To approximate slow-roll inflation, we can take the limit that the slow-roll speed $\dot{\phi}$ and Hubble parameter H are constant, so the time dependence of z comes primarily from $a(\tau)$. With a constant Hubble parameter, the conformal time is $\tau = -\frac{1}{aH}$ (with $\tau = 0$ corresponding to the end of inflation). Thus, in this limit:

$$\frac{z''}{z} = \frac{a''}{a} = \frac{2}{\tau^2} \quad \Rightarrow v''_{\boldsymbol{k}} + \left(k^2 - \frac{2}{\tau^2}\right)v_{\boldsymbol{k}} = 0$$

which can be solved exactly, with the boundary conditions from the normalization of $v_{\boldsymbol{k}}$ and choice of vacuum, yielding:

$$v_k = \frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right)$$

so the power spectrum of the field v can be found:

$$\langle \hat{v}_{\boldsymbol{k}} \hat{v}_{\boldsymbol{k}} \rangle = (2\pi)^3 |v_k|^2 = \frac{(2\pi)^3}{k^3} \frac{a^2 H^2}{2} (1 + k^2 \tau^2)$$

When the wavelength of the mode becomes superhorizon $\frac{a}{k} \gg \frac{1}{H} \Rightarrow |k\tau| \ll 1$ the power approaches a constant:

$$\langle \hat{v}_{\boldsymbol{k}} \hat{v}_{\boldsymbol{k}} \rangle = \frac{(2\pi)^3}{k^3} \frac{a^2 H^2}{2}$$

Changing variables back to the curvature perturbation,

$$\langle \hat{\mathcal{R}}_{\boldsymbol{k}} \hat{\mathcal{R}}_{\boldsymbol{k}} \rangle = \frac{(2\pi)^3}{k^3} \frac{H^4}{2\dot{\phi}^2}$$

This power was derived after horizon crossing for inflation with constant $\dot{\phi}$ and H. However, during slow-roll inflation, these quantities vary slowly. Modes approach constant power after horizon crossing, but they exit horizon at different times – the constant power that the mode approaches depends on the inflationary conditions around the time of its horizon crossing. Then, the power that each mode acquires after its horizon crossing can be found by evaluating the above expressions using $\dot{\phi}$ and H evaluated at horizon crossing for that mode, $a_* H_* = k$:

$$\langle \hat{\mathcal{R}}_{\boldsymbol{k}} \hat{\mathcal{R}}_{\boldsymbol{k}} \rangle = \frac{(2\pi)^3}{k^3} \frac{H_*^4}{2\dot{\phi}_*^2}$$

Or expressed as a dimensionless power spectrum:

$$\Delta_{\mathcal{R}}^2(k) = \frac{k^3}{2\pi^2} \frac{\langle \hat{\mathcal{R}}_{\boldsymbol{k}} \hat{\mathcal{R}}_{\boldsymbol{k}} \rangle}{(2\pi)^3} = \frac{H_*^4}{4\pi^2 \dot{\phi}_*^2}$$

This power is nearly scale-invariant because $\dot{\phi}$ and H change slowly during inflation, and its amplitude depends on those inflationary parameters. Because the curvature perturbation is conserved for adiabatic superhorizon modes, this power in $\mathcal{R}$ is conserved until these modes re-enter horizon. Thus, this inflationary prediction can be carried into times when standard Big Bang cosmology holds, allowing direct comparison with observations.

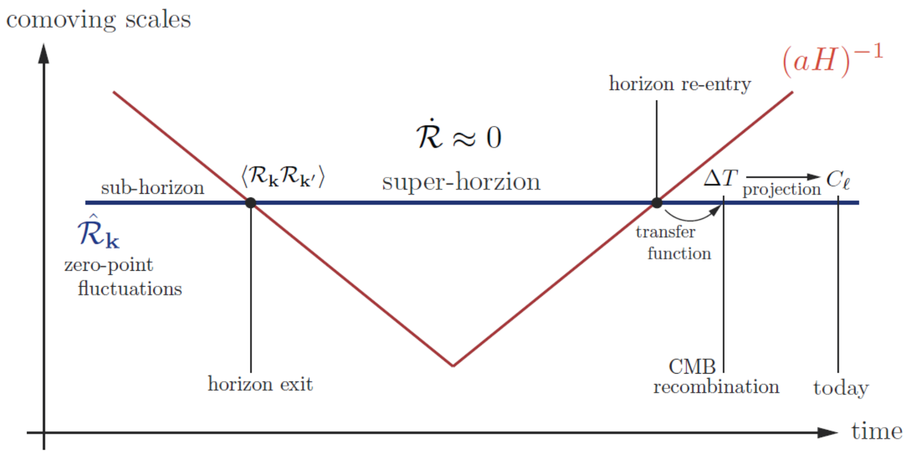

Figure 1. Creation and evolution of perturbations in the inflationary universe. Reprinted from (Baumann, 2009).

Slow-Roll Parameterization

For slow-roll inflation, the spectral tilt of the perturbation spectrum can be found in terms of the slow-roll parameters:

$$\varepsilon = -\frac{\dot{H}}{H^2} \quad \eta = -\frac{\ddot{\phi}}{H\dot{\phi}} = \varepsilon + \frac{1}{2\varepsilon}\frac{d\varepsilon}{dN}$$

$$n_s - 1 = \frac{d \log \Delta_{\mathcal{R}}^2}{d \log k} = 2\eta - 4\varepsilon = -2\varepsilon + \frac{1}{\varepsilon}\frac{d\varepsilon}{dN}$$

where N is a change of variables from time and is equal to the number of e-folds before the end of inflation. Since the slow-roll parameters are functions of time, the spectral tilt weakly depends on scale, but it is often evaluated at a pivot scale close to the scales of perturbations that are observed. During slow-roll, $0 < \varepsilon \ll 1$; because inflation ends when $\varepsilon \sim 1$, ε increases with time and so decreases with N. Then, $n_s - 1 < 0$, predicting a red spectrum: a spectrum with more power at smaller scales.

Primordial gravitational waves are produced by a similar mechanism to the primordial density perturbation, except that gravitational waves are sourced by tensor perturbations rather than scalar perturbations. The amplitude of the gravitational waves is often parameterized in terms of the tensor-to-scalar ratio:

$$r = \frac{\Delta_t^2}{\Delta_s^2} = 16\varepsilon$$

Comparison with the Cosmic Microwave Background

In order to compare these inflationary predictions to the CMB anisotropies, the transfer function from primordial curvature perturbations to these anisotropies must be calculated. This transfer function reflects both the evolution of $\mathcal{R}$ when it re-enters horizon and how the measured anisotropies relate to $\mathcal{R}$. Given the cosmology that holds when the measured modes re-enter horizon, this transfer function can be calculated. This section will not address the transfer function, but instead will outline constraints on inflation from measurements of CMB anisotropies. While constraints on the generic slow-roll parameters ε and η can be made, inflationary models often offer physical justification for specific forms of the inflationary potential $V(\phi)$. Thus, these measurements constrain the parameters of specific models and can even rule them out completely.

(Planck Collaboration, 2013) uses mainly the Planck temperature data and the WMAP CMB polarization data. The temperature anisotropies best constrain the primordial density perturbations, but there is a degeneracy with the optical depth to reionization which is broken by the E mode polarization data. The temperature anisotropies can also constrain the tensor modes. At the pivot scale $k = 0.002\ Mpc^{-1}$, the spectral index for the scalar perturbations is:

$$n_s = 0.9603 \pm 0.0073$$

with exact scale invariance ruled out to more than 5σ, and in agreement with the slow-roll prediction of a red spectrum. The constraint on r is $r < 0.12$ at 95% confidence. These constraints can be used to constrain a variety of inflationary models, not just single field slow-roll inflation.

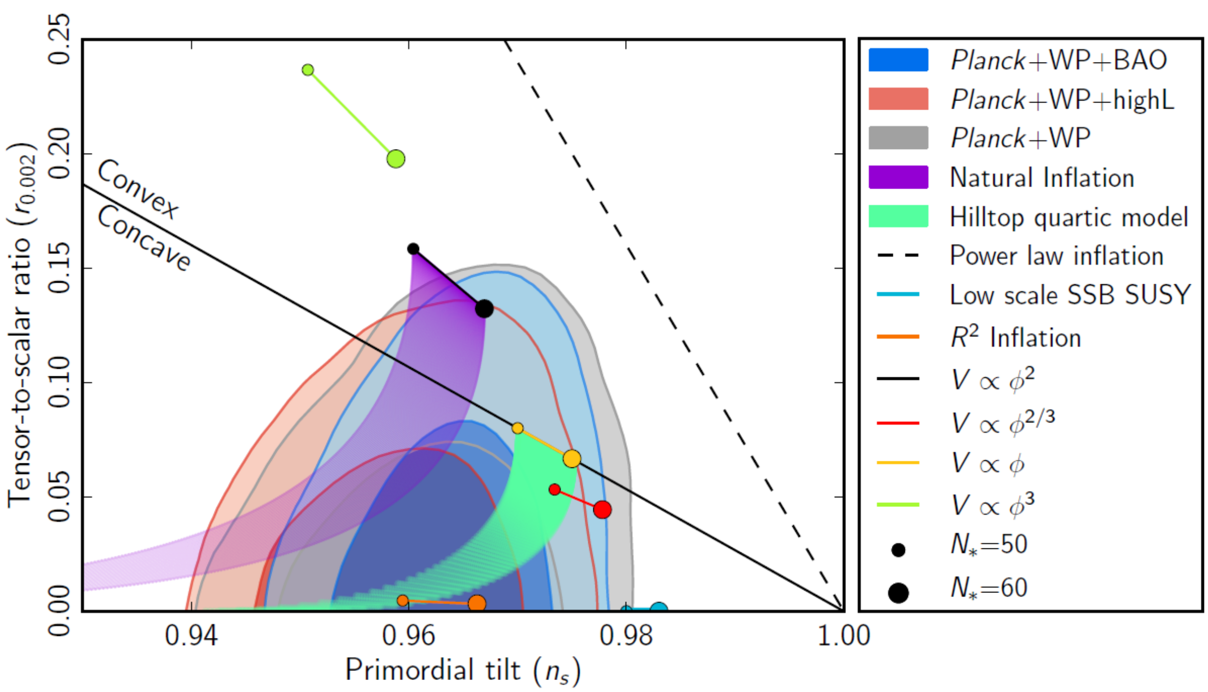

Figure 2. Likelihoods for n_s and r compared to selected inflationary models. Reprinted from (Planck Collaboration, 2013).

(BICEP2 Collaboration, 2014) has claimed the detection of primordial gravitational waves with $r = 0.20^{+0.07}_{-0.05}$ with $r = 0$ ruled out to more than 5σ. The tension with the Planck limit on r can be alleviated if a running on the spectral tilt $\frac{dn_s}{d\log k} = -0.022 \pm 0.010$ is allowed. Running is expected because the slow-roll parameters change over time, but this running is expected to be second-order $\sim\mathcal{O}(\varepsilon^2)$ in single-field slow roll inflation. However, extensions to the LCDM model could also explain this discrepancy. By the Lyth bound (Lyth, 1997), this measurement of the gravitational waves favours large-field inflation, where the inflaton field rolls more than the Planck energy:

$$\frac{\Delta\phi}{M_{Pl}} \sim \left(\frac{r}{0.01}\right)^{\frac{1}{2}}$$

Two prominent large-field inflation models are chaotic inflation (Linde, 1983) and natural inflation (Freese, et al., 1990) (Adams, et al., 1993). In contrast to large-field inflation, small field inflation models with $\Delta\phi \ll M_{Pl}$ often arise in particle physics motivated spontaneous symmetry breaking models.

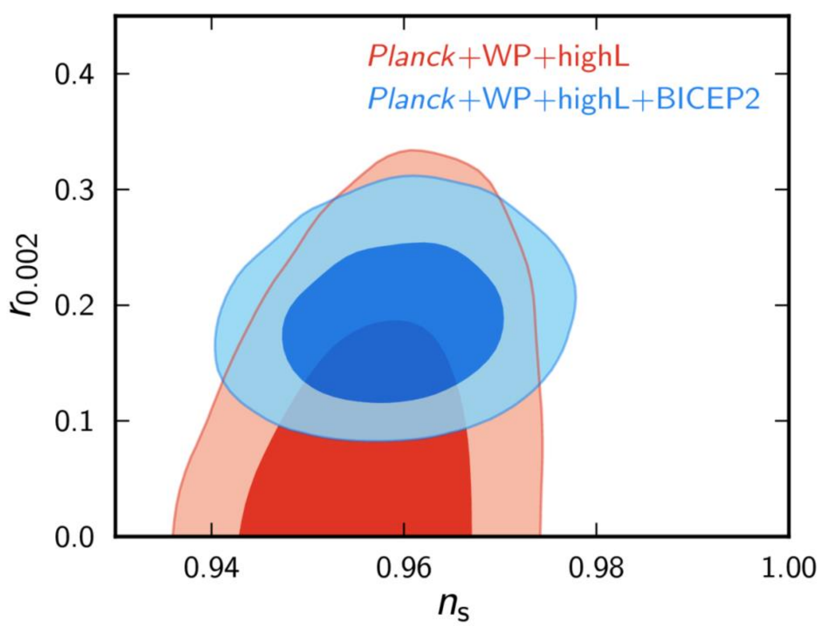

Figure 3. Constraints on r are relaxed when running of the spectral index is allowed. Reprinted from (BICEP2 Collaboration, 2014).

Conclusion

Part of the original motivation for inflation was to explain why the initial conditions of Big Bang cosmology are homogeneous on large scales, but inflation also sources inhomogeneities in the universe from the quantum fluctuations of the scalar field. Inflation allows our observable universe to be completely in causal contact in the past and become homogeneous; during inflation, the inflaton field dominates, so its quantum fluctuations determine the inhomogeneities that persist after inflation. The classical evolution of curvature perturbations during inflation can be determined by perturbing the inflationary action. Quantizing this classical evolution for single field slow-roll inflation reveals that comoving modes acquire a constant power after they cross horizon; that is, perturbations follow the inflationary expansion and are locked in by horizon crossing. Because adiabatic curvature perturbations are conserved on superhorizon scales even after inflation ends, these perturbations are carried forward into the Big Bang cosmology.

Measurements of the anisotropies in the cosmic microwave background have confirmed the predictions of single field slow-roll inflation. A nearly scale-invariant red spectrum of adiabatic scalar perturbations is observed, in agreement with single field slow-roll inflation. Measurements of the tensor mode amplitude and of the running of the scalar spectral tilt will further discriminate between different inflationary potentials in single field slow-roll inflation, or perhaps favour more complicated inflationary theories over single field slow-roll inflation.

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